Regression equations

$$
\begin{aligned}
E\left(D/Y|hieng\right) &= \exp(\beta_0 + \beta_1 I(hieng = high)) \\
\lambda(t|hieng) &= \exp(\beta_0 + \beta_1 I(hieng = high)) \\
\ln(\lambda(t|hieng)) &= \beta_0 + \beta_1 I(hieng = high)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\lambda(t|hieng = high)}{\lambda(t|hieng = low)} &= \exp(\beta_0 + \beta_1)/\exp(\beta_0) \\
&= \exp(\beta_1)
\end{aligned}
$$

$$
\begin{aligned}
\ln(\lambda(t|hieng = high)) - ln(\lambda(t|hieng = low)) &= \beta_0 + \beta_1 - \beta_0 \\
&= \beta_1
\end{aligned}
$$

Regression equations

$$\ln(\lambda(t|energy)) = \beta_0 + \beta_1 I(energy \in [2500, 3000)) + \beta_2 I(energy \in [3000, 4500))$$

$$\begin{aligned}
\lambda(t|energy) &= \exp(\beta_0 + \beta_1 energy) \\
\frac{\lambda(t|energy = x+1)}{\lambda(t|energy = x)} &= \exp(\beta_0 + \beta_1(x+1))/\exp(\beta_0 + \beta_1 x) \\
&= \exp(\beta_1)
\end{aligned}$$

$$\ln(\lambda(t|energy)) = \beta_0 + \beta_1 I(energy/100)$$

Regression equations

$$\ln(\lambda(t|hieng, job)) = \beta_0 + \beta_1 I(hieng = high) + \beta_2 I(job = 2) + \beta_3 I(job = 3)$$

$$\ln(\lambda(t|hieng = high, job)) - \ln(\lambda(t|hieng = low, job)) = \beta_1$$

$$\begin{aligned}\ln(\lambda(t|hieng, job)) = \beta_0 + \beta_1 I(hieng = high) + \beta_2 I(job = 2) + \beta_3 I(job = 3) + \\ \beta_4 I(hieng = high \& job = 2) + \beta_5 I(hieng = high \& job = 3)\end{aligned}$$

$$\begin{aligned}\ln(\lambda(t|hieng = high, job)) - \ln(\lambda(t|hieng = low, job)) \\ = \beta_1 + \beta_4 I(job = 2) + \beta_5 I(job = 3)\end{aligned}$$

Regression equations

$$\begin{aligned}\ln(\lambda(t|hieng, job)) = \beta_1 I(job = 1) + \beta_2 I(job = 2) + \beta_3 I(job = 3) + \\ \beta_4 I(hieng = high \& job = 1) + \\ \beta_5 I(hieng = high \& job = 2) + \\ \beta_6 I(hieng = high \& job = 3)\end{aligned}$$

$$\begin{aligned}\ln(\lambda(t|hieng = high, job)) - \ln(\lambda(t|hieng = low, job)) \\ = \beta_4 I(job = 1) + \beta_5 I(job = 2) + \beta_6 I(job = 3)\end{aligned}$$

Risk calculations

For a constant hazard, we can calculate the risk to time t using

$$\hat{Risk}(t) = 1 - \exp(-\hat{\lambda} t)$$

Let the 95% confidence interval for $\hat{\lambda}$ be $(\hat{\lambda}_l, \hat{\lambda}_u)$. Then the 95% confidence interval for the risk is

$$\left(1 - \exp(-\hat{\lambda}_l t), 1 - \exp(-\hat{\lambda}_u t)\right)$$

Likelihoods

Cox regression:

$$L(\beta) = \prod_{j=1}^{J} \left[\frac{\exp(x_j \beta)}{\sum_{i \in R_j} \exp(x_i \beta)} \right]$$

Likelihoods

Cox regression:

$$L(\beta) = \prod_{j=1}^{J} \left[\frac{\exp(x_j \beta)}{\sum_{i \in R_j} \exp(x_i \beta)} \right]$$

Nested case-control study:

$$L(\beta) = \prod_{j=1}^{J} \left[\frac{\exp(x_j \beta)}{\sum_{i \in R_j^*} \exp(x_i \beta)} \right]$$