# Biostat III Examination 2016 Answers 

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May 17, 2016

## Set-up

```
. global folder 1
```

. set linesize 80

## Commentary

In the following answers, the code and full Stata output are provided together with the answers. The full Stata output was not required in the given answers, but is given here to show how the answers were found.

Some brief comments are warranted on presentation. First, when the question asks for specific results, then those results should be presented separately in text, rather than only presenting the output from the statistical package. Second, the choice of non-proportional fonts makes it difficult to read output from the statistical package. Third, using colours in the graphics makes it difficult to discern which line is which in black-and-white printout. I suggest that using scheme (s2mono) would be useful for graphics in Stata.

## Part 1

## Question 1

We read in the dataset:

```
. import delimited "http://biostat3.net/download/exams/2016/$folder/incidence.c
> sv", clear
(6 vars, 360 obs)
. egen agecat = cut(age), at (40, 50, 60, 70, 80, 90)
```

We then fit a Poisson regression with the number of lung cancer cases at the outcome (first argument), with the person-time of exposure as the exposure option. We include attained age as a linear, continuous effect in each model.

```
. poisson lc sex age, exposure(pt) nolog irr
```

| Poisson regression | Number of obs | $=$ | 360 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (2) | $=$ | 493.81 |
| Log likelihood $=-837.39175$ | Prob > chi2 | $=$ | 0.0000 |
|  | Pseudo R2 | $=$ | 0.2277 |


| lc \| | IRR | Std. Err. | $z$ | P>\|z| | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| sex \| | 2.303417 | .2217275 | 8.67 | 0.000 | 1.907373 | 2.781694 |
| age \| | 1.090729 | .0045982 | 20.60 | 0.000 | 1.081754 | 1.099779 |
| cons \| | $2.13 e-06$ | $5.87 e-07$ | -47.32 | 0.000 | $1.24 e-06$ | $3.65 e-06$ |

. poisson lc smoking age, exposure(pt) nolog irr

| Poisson regression | Number of obs | $=$ | 360 |
| :--- | :--- | :--- | :--- |
|  | LR chi2 (2) | $=$ | 1172.97 |
|  | Prob $>$ chi2 | $=$ | 0.0000 |
| Log likelihood $=-497.81498$ | Pseudo R2 | $=$ | 0.5409 |


| lc \| | IRR | Std. Err. | z | $p>\|z\|$ | [95\% Conf. Interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| smoking \| | 14.59096 | 1.666623 | 23.47 | 0.000 | 11.66 |  | 18.25201 |
| age \| | 1.096176 | . 0046727 | 21.54 | 0.000 | 1.0870 |  | 1.105373 |
| _cons \| | 6.24e-07 | $1.80 \mathrm{e}-07$ | -49.56 | 0.000 | 3.55 e |  | $1.10 \mathrm{e}-06$ |
| $\ln (\mathrm{pt})$ \| | 1 | (exposure) |  |  |  |  |  |
| . poisson lc asbestos age, exposure(pt) nolog irr |  |  |  |  |  |  |  |
| Poisson regression |  |  |  | Number of obs |  | = | 360 |
|  |  |  |  | LR chi2(2) |  |  | 524.76 |
|  |  |  |  | Pro | chi2 | = | 0.0000 |
| Log likelihood = -821.91845 |  |  |  | Pseudo R2 |  | = | 0.2420 |


| lc \| | IRR | Std. Err | z | $\mathrm{P}>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| asbestos \| | 3.679055 | . 3976635 | 12.05 | 0.000 | 2.976674 | 4.547172 |
| age \| | 1.089761 | . 0045805 | 20.45 | 0.000 | 1.08082 | 1.098776 |
| _cons \| | 3.06e-06 | 8.16e-07 | -47.61 | 0.000 | 1.81e-06 | 5.16e-06 |
| $\ln (\mathrm{pt})$ \| | 1 | (exposure) |  |  |  |  |

The age-adjusted incidence rate ratio for sex is 2.30 ( $95 \%$ confidence interval (CI): 1.91, 2.78). This association is highly significant $(p<0.001)$.

The age-adjusted incidence rate ratio for smoking is 14.59 ( $95 \%$ confidence interval (CI): 11.66, 18.25). This association is highly significant ( $p<0.001$ ). The age-adjusted incidence rate ratio for asbestos is 3.68 ( $95 \%$ confidence interval (CI): 2.98, 4.55). This association is highly significant $(p<$ 0.001).

We could have adjusted for attained age in several other ways, including quintiles or splines. To investigate this, we first use quintiles with sex:

```
. xtile ageQ5 = age, nquantiles(5)
. poisson lc sex i.ageQ5, exposure(pt) nolog irr base
```

Poisson regression

Log likelihood = -852.43355

Number of obs $=360$
LR chi2(5) $=463.73$

Prob > chi2 $=0.0000$
Pseudo R2 $=0.2138$

| lc \| | IRR | Std. Err | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sex | 2.282888 | . 2197354 | 8.58 | 0.000 | 1.8904 | 2.756865 |
| ageQ5 |  |  |  |  |  |  |
| 1 \| | 1 | (base) |  |  |  |  |
| 2 \| | 2.838344 | . 479085 | 6.18 | 0.000 | 2.038874 | 3.951296 |
| 31 | 6.696689 | 1.06818 | 11.92 | 0.000 | 4.89876 | 9.154489 |
| 4 \| | 12.24698 | 1.997027 | 15.36 | 0.000 | 8.896714 | 16.85886 |


| 5 | 17.04721 | 3.612093 | 13.38 | 0.000 | 11.25367 | 25.82334 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| _cons | .0000798 | .0000123 | -61.29 | 0.000 | .000059 | .0001079 |
| ln(pt) | 1 | (exposure) |  |  |  |  |

This shows a very similar point estimate and standard errors to modelling attained age as a linear, continuous effect. We also investigate using restricted cubic splines:
. mkspline ageSpline = age, cubic nknots(4)
poisson lc sex ageSpline*, exposure(pt) nolog irr base
Poisson regression Number of obs $=360$
LR chi2 (4) $=504.80$
Prob > chi2 $=0.0000$
Pseudo R2 $=0.2328$

| lc | IRR | Std. Err. | z | $P>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sex | 2.29398 | . 2208203 | 8.63 | 0.000 | 1.899557 | 2.770301 |
| ageSpline1 | 1.140294 | . 0247602 | 6.05 | 0.000 | 1.092783 | 1.18987 |
| ageSpline2 | . 9210111 | . 0576881 | -1.31 | 0.189 | . 8146092 | 1.041311 |
| ageSpline3 | 1.131983 | . 1996173 | 0.70 | 0.482 | . 801192 | 1.59935 |
| _cons | 2.30e-07 | 2.46e-07 | -14.30 | 0.000 | $2.83 e-08$ | $1.87 e-06$ |
| $\ln$ (pt) | 1 | (exposure) |  |  |  |  |

Again, this shows a very similar point estimate and standard errors to modelling attained age as a linear, continuous effect. I accepted answers using any of quintiles, linear/continuous age, splines or similar functional forms.

In summary, lung cancer incidence is associated with age, sex, asbestos exposure and current smoking exposure.

## Question 2

We now adjust for age, sex, smoking exposure and asbestos exposure in the same model.


```
. est store ModelA
```

This shows clearly that each of attained age, sex, smoking and asbestos exposure are significantly associated with lung cancer incidence ( $p<0.001$ for all adjusted effects). The adjusted rate ratio (RR)
for age was 1.098 ( $95 \%$ CI: $1.088,1.107$ ) per year of age, indicating a rapid rise with increasing age. Males have higher rates of disease even after adjustment for other covariates ( $\mathrm{RR}=1.57,95 \% \mathrm{CI}: 1.29$, 1.90). Smoking is strongly associated with lung cancer incidence ( $\mathrm{RR}=13.67,95 \% \mathrm{CI}: 10.91,17.12$ ). Finally, asbestos exposure has a rate ratio of 3.27 ( $95 \%$ CI: 2.64, 4.05).

Empirical evidence for confounding can be assessed in several ways. First, we can assess whether exposure to smoking and asbestos are associated:

```
. tab smoking asbestos [aw=pt], row
+----------------+
| Key |
|----------------- |
    frequency
| row percentage |
+----------------+
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{asbestos} \\
\hline smoking & 0 & 1 & Total \\
\hline \multirow[t]{2}{*}{0} & 253.95541 & 19.564223 & 273.51963 \\
\hline & 92.85 & 7.15 & 100.00 \\
\hline \multirow[t]{2}{*}{1} & 79.645205 & 6.8351613 & 86.480366 \\
\hline & 92.10 & 7.90 & 100.00 \\
\hline \multirow[t]{2}{*}{Total} & 333.60062 & 26.399385 & 360 \\
\hline & 92.67 & 7.33 & 100.00 \\
\hline
\end{tabular}
```

We see that exposure to asbestos is reasonably similar between never smokers ( $7.2 \%$ ) and current smokers ( $7.9 \%$ ), suggesting that asbestos and smoking exposure are not associated, and therefore are unlikely to be confounded. We are not able to undertake a formal statistical test with these weighted data.

Second, we can assess whether the estimated associations between lung cancer incidence and each of smoking and asbestos change after an adjustment for other covariates.

Comparing the linear age-adjusted model with a model with attained age, smoking and asbestos, we see that the rate ratio for asbestos changed from 3.70 to 3.51 ( $5 \%$ reduction), and the rate ratio for smoking changed from 14.59 to 14.42 ( $1 \%$ reduction). Again, there is limited evidence for confounding between smoking and asbestos.

## Question 3

(a)

A regression model formula is

$$
\begin{gathered}
\log (\lambda(t \mid x))=\beta_{0}+\beta_{1} \text { age }+\beta_{2} I(\operatorname{sex}=1)+\beta_{3} I(\text { smoking }=1)+\beta_{4} I(\operatorname{asbestos}=1)+ \\
\beta_{5} I(\text { smoking }=1 \& \text { asbestos }=1)
\end{gathered}
$$

where $\lambda(t \mid x)$ is the rate at attained age $t$ given covariates $x$ (including sex, smoking and asbestos), with coefficients $\beta_{0}, \beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}$ and $\beta_{5}$, and $I($ test ) is 1 if the test is true and 0 if the test is false.
(b)

We now fit the interaction model:

| • poisson lc age sex smoking\#\#asbestos, exposure(pt) nolog irr |  |  |
| :--- | :--- | :--- |
| Poisson regression | Number of obs | $=$ |
|  | LR chi2 $(5)$ | $=$ |


| Log likelihood | -431.4486 |  |  | Pseudo R2 |  | 0.6021 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| lc \| | IRR | Std. Err. | z | $\mathrm{P}>\|\mathrm{z}\|$ | [95\% Con | Interval] |
| age \| | 1.09749 | . 0046902 | 21.77 | 0.000 | 1.088335 | 1.106721 |
| sex \| | 1.563397 | . 1523218 | 4.59 | 0.000 | 1.291626 | 1.892352 |
| 1.smoking \| | 15.95602 | 2.166946 | 20.40 | 0.000 | 12.22714 | 20.82207 |
| 1.asbestos \| | 5.354885 | 1.196287 | 7.51 | 0.000 | 3.456136 | 8.296779 |
| \| |  |  |  |  |  |  |
| smoking\#\| |  |  |  |  |  |  |
| asbestos \| |  |  |  |  |  |  |
| 11 \| | . 5353407 | . 1366072 | -2.45 | 0.014 | . 3246551 | . 8827512 |
| 1 |  |  |  |  |  |  |
| _cons \| | $3.47 \mathrm{e}-07$ | 1.05e-07 | -48.99 | 0.000 | $1.92 \mathrm{e}-07$ | $6.29 \mathrm{e}-07$ |
| $\ln (\mathrm{pt})$ \| | 1 | (exposure) |  |  |  |  |
| . est store ModelB |  |  |  |  |  |  |
| . lrtest ModelA ModelB |  |  |  |  |  |  |
| Likelihood-ratio test |  |  |  | LR | chi2(1) = | 5.71 |
| (Assumption: ModelA nested in ModelB) |  |  |  |  | > chi2 | 0.0168 |

Comparing Model A with Model B, we see that there is good evidence for a statistical interaction on a multiplicative scale. First, we note that the Wald test for the interaction term has a p-value of 0.014 . Second, we see that the likelihood ratio test is also highly significant, with $p=0.017$. We can re-express the rate ratios for different combinations of smoking and asbestos:

```
. poisson lc age sex smoking#asbestos, exposure(pt) nolog irr
```

| Poisson regression | Number of obs | $=$ | 360 |
| :--- | :--- | :--- | ---: |
|  | LR chi2 $(5)$ | $=$ | 1305.70 |
| Log likelihood $=-431.44866$ | Prob $>$ chi2 | $=$ | 0.0000 |
|  | Pseudo R2 | $=$ | 0.6021 |


| lc \| | IRR | Std. Err. | z | $\mathrm{P}>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| age \| | 1.09749 | . 0046902 | 21.77 | 0.000 | 1.088335 | 1.106721 |
| sex \| | 1.563397 | . 1523218 | 4.59 | 0.000 | 1.291626 | 1.892352 |
| 1 |  |  |  |  |  |  |
| smoking\#\| |  |  |  |  |  |  |
| asbestos \| |  |  |  |  |  |  |
| 01 \| | 5.354885 | 1.196287 | 7.51 | 0.000 | 3.456136 | 8.296779 |
| 10 \| | 15.95602 | 2.166946 | 20.40 | 0.000 | 12.22714 | 20.82207 |
| 11 \| | 45.74092 | 7.678061 | 22.77 | 0.000 | 32.91716 | 63.56051 |
| I |  |  |  |  |  |  |
| _cons \| | $3.47 \mathrm{e}-07$ | 1.05e-07 | -48.99 | 0.000 | $1.92 \mathrm{e}-07$ | $6.29 \mathrm{e}-07$ |
| $\ln (\mathrm{pt})$ \| | 1 | (exposure) |  |  |  |  |

This shows that lung cancer incidence rate ratio for exposure to both asbestos and smoking is 45.74 ( $95 \%$ CI: $32.92,63,56$ ) compared with no exposure to both risk factors. However, the effect of both risk factors is significantly less than multiplicative.

## (c)

From Model B, we can calculate the incidence rate for a males aged 62 years who has been exposed to asbestos and is a current smoker using several approaches. By hand, we can calculate the rate as
$3.47 e-07 \times 1.09749^{62} \times 1.563397 \times 45.74092 \approx 0.0079$ per person-year. To calculate the confidence interval, we need to take account of the covariance terms, which is best done using tools provided by each statistical package. Using the lincom command:

```
. quietly poisson lc age sex smoking##asbestos, exposure(pt) nolog irr
. lincom sex + 1.smoking + 1.asbestos + 1.smoking#1.asbestos + 62*age + _cons,
> irr
( 1) 62*[lc]age + [lc]sex + [lc]1.smoking + [lc]1.asbestos +
    [lc]1.smoking#1.asbestos + [lc]_cons = 0
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline lc | & IRR & Std. Err. & z & P> \(\mid\) z \(\mid\) & [95\% Conf & Interval] \\
\hline (1) | & . 0079392 & . 0008947 & -42.91 & 0.000 & . 0063659 & . 0099015 \\
\hline
\end{tabular}
```

This shows that the incidence rate is 7.94 ( $95 \% \mathrm{CI}: 6.37,9.90$ ) per 1000 person-years.
We can do the same analysis using the margins and predict commands.

## Part 2

## Question 4

We read in the data using the following:

```
. display "Folder = $folder"
Folder = 1
. import delimited "http://biostat3.net/download/exams/2016/$folder/survival.cs
> v", clear
(8 vars, 496 obs)
```

(a)

This question is equivalent to completing Table 1 for a randomised controlled trial to assess whether randomisation led to balanced covariates. We use simple tests to assess whether treatment assignment varies substantially by age at diagnosis, sex, smoking exposure and asbestos exposure.

For age at diagnosis, we can use either a t-test or a non-parametric test:

```
. ttest age, by(tx)
```

Two-sample t test with equal variances

| Group | Obs | Mean | Std. Err. | Std. Dev | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 237 | 63.19049 | . 6275862 | 9.661567 | 61.9541 | 64.42687 |
| 1 | 259 | 62.27185 | . 6214458 | 10.00122 | 61.0481 | 63.4956 |
| combined | 496 | 62.7108 | . 441883 | 9.841202 | 61.8426 | 63.57899 |
| diff |  | . 9186377 | . 8845665 |  | -. 8193388 | 2.656614 |
| diff | (0) | mean(1) |  |  |  | 1.0385 |
| Ho: diff |  |  |  | degre | of freedom | 494 |
| Ha: d |  | Ha: diff $!=0$ |  |  | Ha: diff > 0 |  |
| $\operatorname{Pr}(\mathrm{T}<\mathrm{t}$ <br> . ranksum | $\begin{aligned} & 8502 \\ & \text { by (t } \end{aligned}$ | $\operatorname{Pr}(\|T\|>\|t\|)=0.2995$ |  |  | $\operatorname{Pr}(\mathrm{T}>\mathrm{t})=0.1498$ |  |

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

| tx 1 | obs | nk sum | expected |
| :---: | :---: | :---: | :---: |
| 0 \| | 237 | 60434 | 58894.5 |
| 1 \| | 259 | 62822 | 64361.5 |
| combined \| | 496 | 123256 | 123256 |
| unadjusted variance | 2542279.25 |  |  |
| adjustment for ties | 0.00 |  |  |
| adjusted variance | 2542279.25 |  |  |

```
Ho: age(tx==0) = age(tx==1)
    z = 0.966
    Prob > |z|=0.3343
```

We find some evidence that age differs by treatment modality ( $p=0.028$ for the t-test and $p=0.050$ for the Wilcoxon test), where the lung cancer patients randomised to conventional therapy are slightly older than patients randomised to chemotherapy+radiotherapy. For the other variables:
. tab tx sex, chi row

```
|----------------+
```

|  | sex |  |  |
| :---: | :---: | :---: | :---: |
| tx | 0 | 1 | Total |
| 0 | 88 | 149 | 237 |
|  | 37.13 | 62.87 | 100.00 |
| 1 | 71 | 188 | 259 |
|  | 27.41 | 72.59 | 100.00 |
| Total | 159 | 337 | 496 |
|  | 32.06 | 67.94 | 100.00 |

    Pearson chi2(1) \(=5.3657 \quad \operatorname{Pr}=0.021\)
    . tab tx smoking, chi row




We again find some evidence that randomisation varied by sex, where there are the proportion of males n those randomised to conventional is lower than for those randomised to chemotherapy + radiotherapy ( $p=0.002$ ). There is no evidence for imbalance by smoking and asbestos exposure.

In summary, further analyses are potentially confounded by age and sex and we should consider adjusting for those variables in the survival analysis.
(b)

We stset the data using time since diagnosis as the primary time scale and then plot the Kaplan-Meier curves

```
. stset tsurv, failure(event) id(id)
```

id: id
failure event: event != 0 \& event < .
obs. time interval: (tsurv[_n-1], tsurv]
exit on or before: failure

496 total observations
0 exclusions
----------------------------------------------- 496 observations remaining, representing
496 subjects
435 failures in single-failure-per-subject data
530.7275 total analysis time at risk and under observation at risk from $\mathrm{t}=0$

```
. sts graph, by(tx) name(km1, replace) scheme(s2mono)
    failure _d: event
    analysis time _t: tsurv
        id: id
. graph export exam_2016_km1.eps, name(km1) replace
(file exam_2016_km1.eps written in EPS format)
. * the following line is only needed on Linux
. !! convert -density 300 exam_2016_km1.eps exam_2016_km1.png
. sts test tx
    failure _d: event
    analysis time _t: tsurv
                        id: id
```

Log-rank test for equality of survivor functions


| Time | Beg. <br> Total | Fail | Survivor <br> Function | Std. <br> Error | [95\% Conf. Int.] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{tx}=0$ |  |  |  |  |  |  |
| 1 | 87 | 145 | 0.3774 | 0.0319 | 0.3151 | 0.4395 |
| 2 | 62 | 25 | 0.2677 | 0.0292 | 0.2122 | 0.3261 |
| 3 | 46 | 13 | 0.2095 | 0.0270 | 0.1593 | 0.2645 |
| 4 | 34 | 11 | 0.1571 | 0.0244 | 0.1129 | 0.2081 |
| 5 | 28 | 4 | 0.1375 | 0.0233 | 0.0959 | 0.1866 |
| $\mathrm{tx}=1$ |  |  |  |  |  |  |
| 1 | 55 | 203 | 0.2136 | 0.0256 | 0.1658 | 0.2656 |
| 2 | 42 | 13 | 0.1622 | 0.0231 | 0.1200 | 0.2100 |
| 3 | 29 | 11 | 0.1174 | 0.0203 | 0.0813 | 0.1606 |
| 4 | 20 | 9 | 0.0797 | 0.0172 | 0.0502 | 0.1177 |
| 5 | 17 | 1 | 0.0752 | 0.0168 | 0.0467 | 0.1126 |

Note: survivor function is calculated over full data and evaluated at indicated times; it is not calculated from aggregates shown at left.

Kaplan-Meier survival estimates


The Kaplan-Meier curves show that survival is poor for lung cancer patients, with fewer than $25 \%$ of patients surviving to 5 years. We also see that treatment with chemotherapy+radiotherapy leads to more deaths soon after diagnosis. It is unclear whether the rates are different after one year.

Although not specifically asked for, we also (i) used the log-rank test to compare the curves, finding strong evidence for a difference $(p=0.0001)$ and (ii) estimated survival to five years, where $14 \%(95 \%$ CI: 10,19 ) survived for those on conventional treatment and $8 \% ~(95 \%$ CI: 5,11$)$ survived for those on chemotherapy+radiotherapy.

## Question 5

Based on Question 4 (a), we first investigated whether age and sex were associated with survival and hence would be potential confounders:



Adjusting for treatment modality, there is no evidence that either age or sex are associated with survival, with Wald test p-values of 0.60 for both age and sex. Furthermore, fitting a Cox regression models with and without age and sex suggest that the effect of treatment modality is insensitive to inclusion of age and sex in the model. The hazard ratio for chemotherapy+radiotherapy compared with conventional therapy is 1.47 ( $95 \%$ CI: $1.22,1.78$ ), suggesting that the average hazard ratio for chemotherapy + radiotherapy is high over the five-year period.

For the time scale, we have initially used time since cancer diagnosis. There is a strong association between time since diagnosis and survival, suggesting that this is the best choice of primary time scale. Moreover, there is a suggestion of non-proportional hazards, with a higher rate ratio in the first year than for the later years. We could investigate using attained age as the primary time scale, but then we would need to finely model for the time since diagnosis, which would require modelling two time scales. For simplicity, we propose using time since diagnosis as the primary time scale.

## Question 6

## (i)

For an analysis of scaled Schoenfeld residuals, we use:

```
. estat phtest, detail
    Test of proportional-hazards assumption
    Time: Time
```



```
. estat phtest, plot(tx) name(phtest, replace) scheme(s2mono)
. graph export exam_2016_phtest.eps, name(phtest) replace
(file exam_2016_phtest.eps written in EPS format)
. * the following line is only needed on Linux
. !! convert -density 300 exam_2016_phtest.eps exam_2016_phtest.png
```

Test of PH Assumption


This shows that there is some evidence $(p=0.10)$ that the hazard ratio decreases with increasing time since diagnosis: the scaled residuals and linear time have a correlation of -0.08 . From the plot of the scaled residuals and time, we see the running mean smoother dips early in the follow-up period and then is flat or very slightly declining. Given the number of events that are early in the period, we could also test using a log-transformation for time since diagnosis:

```
. estat phtest, detail log
    Test of proportional-hazards assumption
    Time: Log(t)
\begin{tabular}{|c|c|c|c|c|}
\hline & rho & chi2 & df & Prob>chi2 \\
\hline tx & -0.10739 & 4.91 & 1 & 0.0267 \\
\hline global test & & 4.91 & 1 & 0.0267 \\
\hline
\end{tabular}
. estat phtest, log plot(tx) name(phtestlog, replace) scheme(s2mono)
. graph export exam_2016_phtestlog.eps, name(phtestlog) replace
(file exam_2016_phtestlog.eps written in EPS format)
. * the following line is only needed on Linux
. !! convert -density 300 exam_2016_phtestlog.eps exam_2016_phtestlog.png
```

Running mean smoother


This transformation indicates a stronger association than that for untransformed time ( $p=0.027$ ), with evidence for linearity in the hazard ratio on the log-time scale.
(ii)

We can test for piecewise-constant hazard ratios by splitting by time and fitting for an interaction. In the following, the " $c$ " prefix indicates a continuous variable, while the " $i$ " prefix indicates a factor variable.

```
. quietly import delimited "http://biostat3.net/download/exams/2016/$folder/sur
> vival.csv", clear
. quietly stset tsurv, fail(event) id(id)
. stsplit timeband, at(0, 1, max)
(140 observations (episodes) created)
. stcox sex i.tx##i.timeband, nolog
    failure _d: event
    analysis time _t: tsurv
        id: id
Cox regression -- no ties
No. of subjects = 496 Number of obs = 636
No. of failures = 435
Time at risk = 530.7275306
Log likelihood = -2368.0965 Prob > chi2 = 0.0001
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|r|}{_t | Haz. Ratio} & Std. Err. & Z & \(P>|z|\) & \multicolumn{2}{|l|}{[95\% Conf. Interval]} \\
\hline sex & . 9535614 & . 0984221 & -0.46 & 0.645 & . 7789179 & 1.167362 \\
\hline 1.tx & 1.636955 & . 1794447 & 4.50 & 0.000 & 1.320465 & 2.0293 \\
\hline 1.timeband & 7.392735 & & & & & \\
\hline
\end{tabular}
```

```
    11 | . 6086022 . 1494051 -2.02 0.043 . 3761602 . 9846777
. stcox tx sex c.tx#c.timeband, nolog
    failure _d: event
    analysis time _t: tsurv
        id: id
Cox regression -- no ties
No. of subjects = 496 Number of obs = 636
No. of failures = 435
Time at risk = 530.7275306
LR chi2(3) = 20.57
Log likelihood = -2368.0965 Prob > chi2 = 0.0001
\begin{tabular}{rlllllll} 
_t | Haz. Ratio & Std. Err. & \(z\) & P>|z| & [95\% Conf. Interval] \\
tx | & 1.636955 & .1794447 & 4.50 & 0.000 & 1.320465 & 2.0293 \\
sex | & .9535614 & .0984221 & -0.46 & 0.645 & .7789179 & 1.167362 \\
c.tx\# | & & & & & & \\
c.timeband | & .6086022 & .1494051 & -2.02 & 0.043 & .3761602 & .9846777
\end{tabular}
```

```
. stcox c.tx#i.timeband, nolog
```

. stcox c.tx\#i.timeband, nolog
failure _d: event
failure _d: event
analysis time _t: tsurv
analysis time _t: tsurv
id: id
id: id
Cox regression -- no ties
Cox regression -- no ties
No. of subjects = 496 Number of obs = 636
No. of subjects = 496 Number of obs = 636
No. of failures = 435
No. of failures = 435
Time at risk = 530.7275306
Time at risk = 530.7275306
LR chi2(2) = 20.36
LR chi2(2) = 20.36
Log likelihood = -2368.202 Prob > chi2 = 0.0000
Log likelihood = -2368.202 Prob > chi2 = 0.0000

| _t \| Haz. Ratio |  | Std. Err. | z | $P>\|z\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| timeband\#c.tx \| |  |  |  |  |  |  |
| 0 | 1.628084 | . 1774136 | 4.47 | 0.000 | 1.314985 | 2.015732 |
| 1 | . 9871128 | . 2169872 | -0.06 | 0.953 | . 641587 | 1.518721 |

```

This model provides some evidence that the hazard ratio is time-dependent ( \(p=0.04\) ). The hazard ratio in the first year is 1.63 ( \(95 \% \mathrm{CI}: 1.31,2.02\) ), while the hazard ratio in the second year is \(0.99(95 \%\) CI: \(0.64,1.52\) ).
(iii)

We can re-fit the model in (ii) using Stata stcox's tvc and texp options:
```

. stcox tx, nolog tvc(tx) texp(_t>=1)

```
```

    failure _d: event
    analysis time _t: tsurv
        id: id
    ```


Again, we find some evidence for a time-dependent hazard ratio. We can model for a time-dependent hazard ratio that depends on time or \(\log\) (time):
```

. stcox tx, nolog tvc(tx) texp(_t)
failure _d: event
analysis time _t: tsurv
id: id

```
Cox regression -- no ties
No. of subjects \(=\quad 496 \quad\) Number of obs \(=\quad 636\)
No. of failures \(=\quad 435\)
Time at risk \(=530.7275306\)
LR chi2(2) = 18.84
Log likelihood = -2368.961 Prob > chi2 \(=0.0001\)


The interpretation of this model is as follows: the hazard ratio at time 0 is 1.64 ( \(95 \% \mathrm{CI}: 1.30,2.06\) ); for each year since diagnosis, the rate tends to decrease by \(1-0.85=15 \%(\mathrm{RR}=0.85,95 \% \mathrm{CI}: 0.69,1.03)\), although this trend is not significant ( \(p=0.10\), as per the Schoenfeld test). We could also model for time-dependence using log-time:
```

. stcox tx, nolog tvc(tx) texp(log(_t))

```
```

    failure _d: event
    analysis time _t: tsurv
    id: id
    ```
Cox regression -- no ties
\begin{tabular}{lrlrl} 
No. of subjects \(=\) & 496 & Number of obs & \(=\) & 636 \\
No. of failures \(=\) & 435 & & \\
Time at risk & \(=530.7275306\) & & & \\
& & LR chi2 (2) & \(=\) & 21.28 \\
Log likelihood \(=\) & -2367.7426 & Prob \(>\) chi2 & \(=\) & 0.0000
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline & \multicolumn{2}{|l|}{_t | Haz. Ratio} & Std. Err. & z & \(\mathrm{P}>|z|\) & \multicolumn{2}{|l|}{[95\% Conf. Interval]} \\
\hline main & | & & & & & & \\
\hline & tx 1 & 1.227843 & . 1550932 & 1.62 & 0.104 & . 9585718 & 1.572755 \\
\hline \multirow[t]{2}{*}{tvc} & 1 & & & & & & \\
\hline & tx 1 & . 9151138 & . 036646 & -2.22 & 0.027 & . 8460352 & . 9898325 \\
\hline
\end{tabular}

Note: variables in tvc equation interacted with \(\log \left(\_t\right)\)
This model can be interpreted as follows: at time equals 1 , the hazard ratio is 1.23 ( \(95 \% \mathrm{CI}\) : 0.96 , 1.57; \(p=0.10\) ); for each unit increase in \(\log (\) time \()\), the hazard ratio is multiplied by a factor of 0.92 ( \(95 \% \mathrm{CI}: 0.85,0.99 ; p=0.027\) ).
(iv)

Using stpm2 with time-dependent hazard ratios, we use a low-dimensional natural spline for the timedependent effect. We use a Wald test to check for time-dependence and plot the time-dependent hazard ratio:
. stpm2 tx, \(\mathrm{df}(4)\) scale(hazard) nolog eform tvc(tx) dftvc(2)
note: delayed entry models are being fitted
Log likelihood = -1124.2991 \(\quad\) Number of obs \(=\quad 636\)

```

chi2( 2) = 4.92

```
```

    Prob > chi2 = 0.0853
    . predict hr, hrnumerator(tx 1) ci
. twoway (rarea hr_lci hr_uci _t if hr_uci<5, sort color(gs12)) (line hr _t if
> hr_uci<5, sort), legend(off) xtitle("Time since diagnosis") ytitle("Hazard ra
> tio") name(hr, replace) scheme(s2mono)
. graph export exam_2016_hr.eps, name(hr) replace
(file exam_2016_hr.eps written in EPS format)
. * the following line is only needed on Linux
. !! convert -density 300 exam_2016_hr.eps exam_2016_hr.png

```


We see that there is limited evidence for time-dependent hazards ( \(p=0.09\) from the Wald test). We also see from the plot that the hazard ratio comparing chemotherapy + radiotherapy with conventional therapy is very high soon after diagnosis and then declines towards 1 , with the confidence interval overlapping with 1 before the end of the first year.

\section*{Question 7}
(a)

Advantages of using Poisson regression for Questions 5-6 include: (i) Poisson regression readily models for multiple time scales, where we could split on attained age and time since diagnosis and then model for main effects and interactions between those time scales and interactions between a time scale and other covariates; (ii) it is simpler to predict rates from Poisson regression, as the analysis is done on that scale.

Disadvantages of using Poisson regression include: (i) the need to split on the time scales, which may increase the size of the computational problem; (ii) the need to specify a functional form for the primary time scale using parametric functions, rather than using Cox regression's non-parametric formulation; (iii) crude time splitting will assume that rates are piece-wise constant, which may not be appropriate; (iv) risk calculations for Poisson regression require that the risk period involves constant rates or numerical integration.
(b)

Assuming that the follow-up time has been split for within one year of diagnosis and from one year of diagnosis, we can model the rate using:
\[
\log (\lambda(t \mid \mathrm{tx}))=\beta_{0}+\beta_{1} I(t<1)+\beta_{2} I(t \geq 1)+\beta_{3} I(\mathrm{tx}=1)+\beta_{4} I(\mathrm{tx}=1 \& t \geq 1)
\]

A better formulation would be to include more time-splits for time since diagnosis. If we let time cuts be represented by \(t_{j}\) where \(t_{0}=0\), then
\[
\log (\lambda(t \mid \mathrm{tx}))=\beta_{0}+\sum_{j} \beta_{j} I\left(t_{j-1}<t \leq t_{j}\right)+\beta_{\mathrm{tx}} I(\mathrm{tx}=1)+\beta_{\mathrm{tx}: t} I(\mathrm{tx}=1 \& t \geq 1)
\]

We could also model using splines. Any similar formulation was accepted, including different formulations for the time-dependent hazard ratios.```

